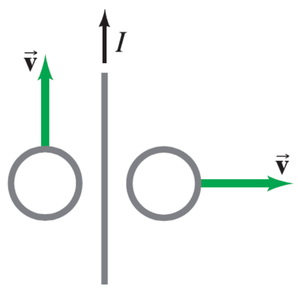
Elektriciteit en magnetisme 2
Instructor: A.M. van den Berg
English version: see pages 3-4

Het is niet noodzakelijk iedere vraag op een apart vel te maken.
Plaats op ieder vel je **naam en S-nummer**
Er zijn **4 vragen** met een totaal aantal punten: 75

SCHRIJF DUIDELIJK

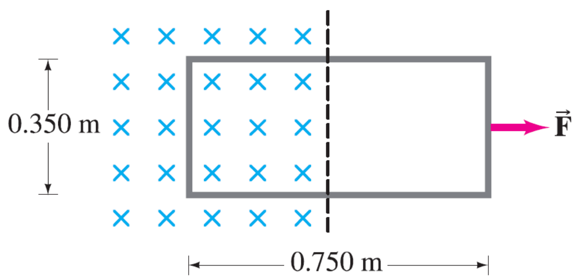
(1) (Totaal 10 punten)
Twee stroomkringen bewegen met een snelheid $\vec{v}$ in de buurt van een oneindige lange draad, waardoor een stroom loopt met een sterkte I; zie de figuur. De stroomkring aan de linkerkant beweegt parallel aan de lange draad; de andere kring beweegt zich loodrecht op de richting van de draad.

(a) (5 punten) Geef de richting aan van de geïnduceerde stroom in ieder van de stroomkringen.
(b) (5 punten) Verklaar je antwoord.



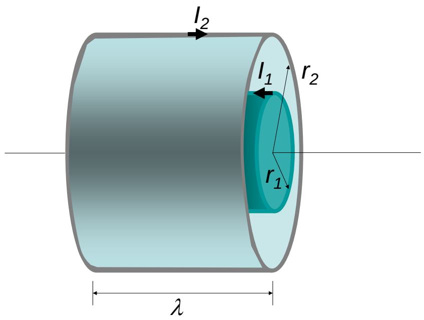
(2) (Totaal 20 punten)
Een enkele rechthoekige stroomkring met afmetingen zoals getoond in de figuur ligt gedeeltelijk in een uniform magneet veld met een sterkte van 0.65 T; richting in de pagina. De totale weerstand van de stroomkring bedraagt 0.28 Ω. De stroomkring wordt met een constante snelheid van 3.40 m s^{-1} naar rechts getrokken. Zwaartekracht en wrijving spelen geen rol in deze opgave.

(a) (5 punten) Neemt de magnetische flux die door de stroomkring wordt omsloten toe of af bij het naar rechts trekken van de stroomkring?
(b) (5 punten) Bereken de geïnduceerde EMF in de stroomkring.
(c) (5 punten) Bereken de grootte van de opgewekte stroom en geef aan welke richting de stroom rond gaat in de stroomkring. Verklaar je keuze.
(d) (5 punten) Bereken de grootte van de kracht, die noodzakelijk is om de stroomkring naar rechts te trekken.



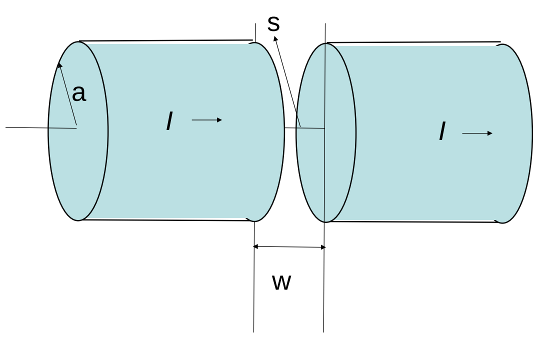
(3) (Totaal 20 punten)
Een coaxiale kabel bestaat uit twee concentrische cylindervormige geleiders. De wanddikte van deze cylinders kan verwaarloosd worden. De straal van de binnengeleider is r_1, die van de buitengeleider is r_2. Door deze geleiders loopt een stroom I, die in beide cylinders even groot is. De richting van deze stroom in de binnengeleider is tegengesteld aan de richting van de stroom in de buitengeleider.

- (a) (5 punten) Bereken de magnetische veld sterkte tussen de beide geleiders.
- (b) (5 punten) Bereken de magnetische flux tussen de beide geleiders over een afstand λ langs de coaxiale kabel. Dus door een oppervlakte A ter grootte: $A = [(r_2 - r_1) \cdot \lambda]$
- (c) (5 punten) Bereken de inductie per lengte eenheid voor deze kabel.
- (d) (5 punten) Bereken de hoeveelheid aan magnetische energie, die opgeslagen is in het magneetveld over de lengte λ van deze kabel en tussen de twee geleiders.



(4) (Totaal 25 punten)
Door een dikke draad met een straal a loopt een stroom I, die uniform verdeeld is over de doorsnede van de draad. Er is een smalle onderbreking in de draad met een breedte $w \ll a$. De spleet (dat is het gebied tussen de twee uiteinden van de onderbroken draad) kan worden beschouwd als de ruimte binnen een parallelle plaat condensator. Zie de figuur.

- (a) (5 punten) Bereken de magnetische veld sterkte in de onderbreking als functie van de radiële afstand s in het gebied $s < a$.
- (b) (10 punten) Neem aan dat op tijdstip $t = 0$ de oppervlakte ladingen $+\sigma$ en $-\sigma$ nul zijn. En dat deze lading continue toeneemt als functie van de tijd; $\sigma = I\ t/\left(\pi a^2\right)$. Bereken de elektrische veld sterkte en de magnetische veldsterkte in de ruimte van de spleet als functie van s en van t.
- (c) (10 punten) Bereken de elektromagnetische energie dichtheid (energie per volume eenheid) u_{EM} en de Poynting vector in de ruimte van de spleet.

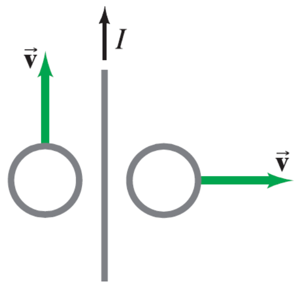


Elektriciteit en magnetisme 2
Instructor: A.M. van den Berg
Nederlandse versie: zie pagina's 1-2

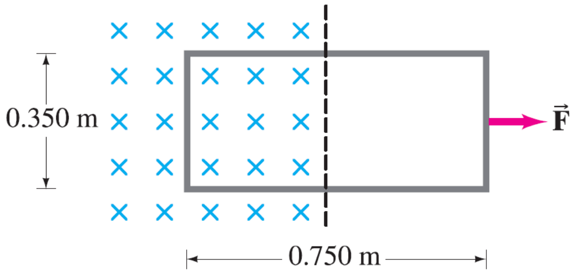
You don't have to use separate sheets for every question.
Write your name and S number on every sheet
There are **4 questions** with a total number of marks: 75

WRITE CLEARLY

(1) (Total 10 punten)
Two loops of wire are moving with a velocity $\vec{v}$ in the vicinity of a very long straight wire which carries a current I; see the figure. The loop at the left-hand side moves parallel to the wire, the one on the right-hand side moves perpendicular to the direction of the wire.
  (a) (5 marks) Give the direction of the induced current in each of the loops.
  (b) (5 marks) Explain your answer.



(2) (Totaal 20 marks)
A single loop with dimensions as shown in the figure is partially located in a uniform magnetic field with strength 0.65 T; the direction of the field points inside the page. The total resistance of the loop is 0.28 Ω. The loop moves with a constant velocity of 3.40 m s^{-1} to the right. Gravitational and friction forces can be neglected.
  (a) (5 marks) Does the magnetic flux enclosed by the loop increase or does it decrease if one moves the loop to the right?
  (b) (5 marks) Calculate the induced EMF in the loop.
  (c) (5 marks) Calculate de induced current in the loop and indicate in which direction the current flows. Explain your choice.
  (d) (5 marks) Calculate the force which is required to pull the loop to the right.



(3) (Total 20 marks)
A coaxial cable is made of two concentric cylindrical shaped conductors. The thickness of the wall of these conductors can be neglected. The radius of the inner conductor is r_1, the one of the outer conductor is r_2. Through these conductors runs a current I which is equal

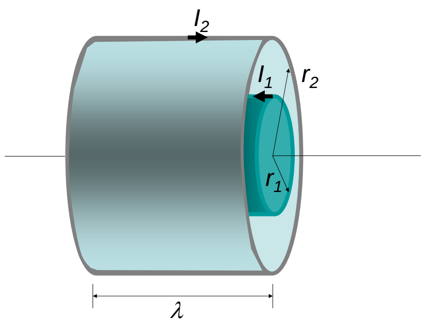
in magnitude for both conductors. The direction of the current in the outer conductor is opposite to that in the inner conductor.

(a) (5 marks) Calculate the magnetic field strength between these two conductors.

(b) (5 marks) Calculate the magnetic flux between these two conductors over a distance with length λ along the coaxial cable. Thus through an area A with dimensions: $A = [(r_2 - r_1) \cdot \lambda]$

(c) (5 marks) Calculate the inductance per unit length of this cable.

(d) (5 marks) Calculate the magnetic energy contained by the magnetic field in the region between the two conductors and over a length λ of this cable.



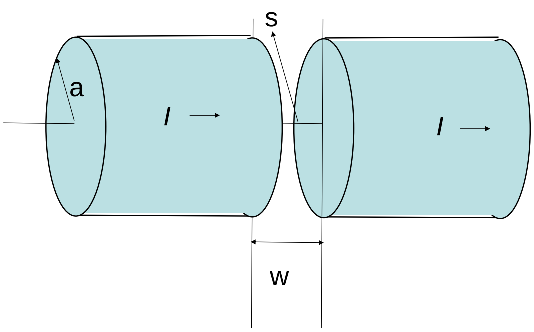
(4) (Total 25 marks)

A fat wire with radius a carries a constant current I, uniformly distributed over its cross section. A narrow gap in the wire, of width $w \ll a$, forms a parallel plate capacitor, as shown in the figure.

(a) (5 marks) Calculate the magnetic field strength in the gap, at a distance $s < a$.

(b) (10 marks) Assume that at time $t = 0$ the surface charges $+\sigma$ en $-\sigma$ are equal to zero. And that these charges increase constant in time; $\sigma = I\, t / \left(\pi a^2\right)$. Calculate the electric and magnetic field strengths in the gap as a function of s and of t.

(c) (10 marks) Calculate the electromagnetic energy density (energy per unit of volume) u_{EM} and the Poynting vector in the gap.



Solutions

(1) Faraday's law tells you that an induced electric field (emf) will be created in case the magnetic flux changes. In the present case each of the two coils maintains its dimension and the angle between the magnetic field vector and the normal to the surfaces enclosed by the coils does not change. Thus the only thing that matters is the strength of the magnetic field vector. This strength decreases as a function of s, being the radial distance from the wire.
The coil which moves parallel to the wire will encompass a magnetic field flux which is constant in time. Therefore, no emf and thus no current will be induced in the coil shown in the left-hand side of the figure.
The other coil is moving away from the wire; in this case the magnetic flux encompassed by the coil will decrease during this movement. According the law of Lenz a current will be induced which counteracts this decreasing magnetic flux. The magnetic field flux through this loop (on the right-hand side) is directed into the page. Therefore, the induced current in this loop will have a clock-wise direction.

(2)
- The loop is pulled to the right. The magnetic field remains constant in magnitude and also the direction between the normal to surface of surface subtended by the coil and the magnetic field vector remains constant. Therefore, the only thing that matters is the area of the loop contained by the magnetic field. This area decreases while pulling the loop to the right; thus the magnetic flux decreases.
- The electromotive force for this case can be calculated as $\varepsilon = -\frac{d\Phi}{dt} = -\frac{d}{dt}\oint \vec{B} \circ d\vec{a}$. As the magnetic field remains constant, we only need to calculate the rate at which the enclosed area changes during the movement. Thus $\frac{d\Phi}{dt} = Bh\frac{dx}{dt} = Bhv$, where h is the width of the coil ($h = 0.35$ m). The magnitude for the emf is thus given as $\varepsilon = Bhv$. Using the values given, we can calculate that $\varepsilon = 0.7735$ V.
- The induced current can be calculated from Ohm's law: $\varepsilon = IR$; thus $I = \frac{\varepsilon}{R} = 2.7625$ A. The magnetic field points into the page and the magnetic flux encompassed by the coil decreases. The induced current counteracts the decreasing magnitude of the magnetic flux. The induced current will run therefore clockwise through the loop.
- The magnetic field B acts on the current in the loop, which runs clockwise through the loop. By the law of Lorentz there will be a magnetic force pointing to the left. To keep the coil moving an equal but opposite force must act on the coil. The magnitude of the magnetic force is given as: $F = I\oint\left(d\vec{\ell} \times \vec{B}\right)$. The forces on the horizontal parts of the coil are equal in strength, but opposite to each other and therefore, their contributions cancel. The vertical bars are inside the magnetic field (left) and outside the magnetic field (right). The only piece of the path integral is therefore along the vertical bar. Because the vertical bar is perpendicular to the magnetic field vector, we find $F = IhB = 0.628$ N.

(3)
- The magnetic field in between the conductors can be calculated using the law of the Ampère: $\oint \vec{B} \circ d\vec{\ell} = \mu_0 I_{encl}$. The enclosed current is the current running through the inner conductor. An Amperian loop can be made by selecting a circular path with radius s, with $r_1 < s < r_2$. Because of symmetry, the magnetic field has only components in the ϕ direction, thus tangential to this Amperian loop. The inner

product between $\vec{B}$ and $d\vec{\ell}$ is just the product $Bd\ell$. Thus $2\pi sB = \mu_0 I_1 = \mu_0 I$. And for the radial distance $r_1 < s < r_2$, the magnetic field is: $B = \dfrac{\mu_0 I}{2\pi s}$.

- The magnetic field is pointing into the direction ϕ. The area A has a length λ and a width $r_2 - r_1$. The normal to this area is also pointing in the direction ϕ and is therefore parallel to $\vec{B}$. The magnetic flux is: $\Phi_B = \int_A \vec{B} \circ d\vec{a} = \int_A B da = \int_{r_1}^{r_2} \lambda B ds$, with $B = \dfrac{\mu_0 I}{2\pi s}$.
  We integrate from $s = r_1$ to $s = r_2$, thus $\Phi_B = \dfrac{\mu_0 I \lambda}{2\pi} \int_{r_1}^{r_2} \dfrac{1}{s} ds = \dfrac{\mu_0 I \lambda}{2\pi} \ln \dfrac{r_2}{r_1}$.
- The inductance is the ratio of the magnetic flux and the current; the inductance per unit length is thus: $\dfrac{L}{\lambda} = \dfrac{\Phi_B}{\lambda I} = \dfrac{\mu_0}{2\pi} \ln \dfrac{r_2}{r_1}$.
- The magnetic energy contained by the magnetic field in the region between the two conductors is given as $U_B = \dfrac{1}{2} L I^2$. As I is given and we have an expression for L, so we find that $U_B = \dfrac{\mu_0 I^2 \lambda}{4\pi} \ln \dfrac{r_2}{r_1}$.
  Alternatively, we can calculate U_B from the volume integral of $\dfrac{1}{2\mu_0} B^2 d\vec{\tau}$, where the volume has length λ and is bounded by the two radii r_1 and r_2.
  Thus $U_B = \int_{r_1}^{r_2} \dfrac{1}{2\mu_0} \left(\dfrac{\mu_0 I}{2\pi s} \right)^2 2\pi \lambda s ds$. This yields the same answer.

(4)

- According to the law of Maxwell and Ampère, the magnetic field strength is given as: $\oint_P \vec{B} \circ d\vec{\ell} = \mu_0 I_{free\ enclosed} + \mu_0 \epsilon_0 \int_S \dfrac{\partial \vec{E}}{\partial t} \circ d\vec{a}$. Make an Amperian loop centered at the axis through the pieces of wire with a radius $s < a$, and halfway between the two "plates". Because of symmetry reasons the magnetic field is in the direction ϕ, thus tangential to the radial direction s. The LHS of this equation therefore is $2\pi sB$. The RHS has two terms, the first from the free current through the Amperian loop with radius s, the second term corresponds to the displacement current density $\vec{J}_d = \epsilon_0 \dfrac{\partial \vec{E}}{\partial t}$. Because there are no free currents through the loop, the first term equals zero.
  For the second term we note that the current density is $\vec{J}_d = \dfrac{I}{2\pi a^2} \hat{z}$.
  Therefore, the equation becomes $2\pi sB = \mu_0 \dfrac{I}{\pi a^2} \pi s^2 = \mu_0 I \dfrac{s^2}{a^2}$.
  Solving for B we find: $\vec{B} = \dfrac{\mu_0 I s}{2\pi a^2} \hat{\phi}$.
- The electric field between the plates is given as $\vec{E} = \dfrac{\sigma}{\epsilon_0} \hat{z}$, where the surface charge density σ is equal to: $\sigma = \dfrac{Q}{\pi a^2} = \dfrac{It}{\pi a^2}$.
  We find $\vec{E} = \dfrac{It}{\pi a^2 \epsilon_0} \hat{z}$.
  As in the previous question, we use Maxwell Ampère to calculate the magnetic field strength from the time derivative of E: $\int_P \vec{B} \circ d\vec{\ell} = \oint_S \mu_0 \epsilon_0 \dfrac{\partial \vec{E}}{\partial t} \circ d\vec{a}$, where the Amperian loop over the path P with radius s (LHS of the equation yielding $B 2\pi s$) and the surface S required to integrate the electric field flux $\vec{E} \circ d\vec{a}$ yielding $E \pi s^2$ (RHS) have

the same radius s!
Thus $\vec{B} = \frac{1}{2\pi s}\mu_0\epsilon_0\frac{I}{\pi\epsilon_0 a^2}\pi s^2\hat{\phi} = \frac{\mu_0 I s}{2\pi a^2}\hat{\phi}$.

- The electromagnetic field density is the energy density PER VOLUME caused by the electric and magnetic fields: $u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)$.
  Using the calculated values for E and B, we find: $u = \frac{\mu_0 I^2}{2\pi^2 a^4}\left[(ct)^2 + (s/2)^2\right]$.
  The Poynting vector is given as: $\vec{S} = \frac{1}{\mu_0}\left(\vec{E}\times\vec{B}\right)$.
  We know the directions of E (direction $\hat{z}$) and of B (direction $\hat{\phi}$); therefore, the Poynting vector is $\vec{S} = \frac{-I^2 ts}{2\pi^2\epsilon_0 a^4}\hat{s}$.